Family of Functions Lesson

Introduction:

Show pictures of family members to illustrate that even though family members are different (in most cases) they have very similar characteristics (DNA).

"Today we are going to investigate a variety of different families of functions. Similar to human beings, mathematical families have similar characteristics within a family. The first mathematical families we are going to investigate are the family of linear functions, or lines."

Write the equation $y = x$ on the board/overhead.

Explain to students that:

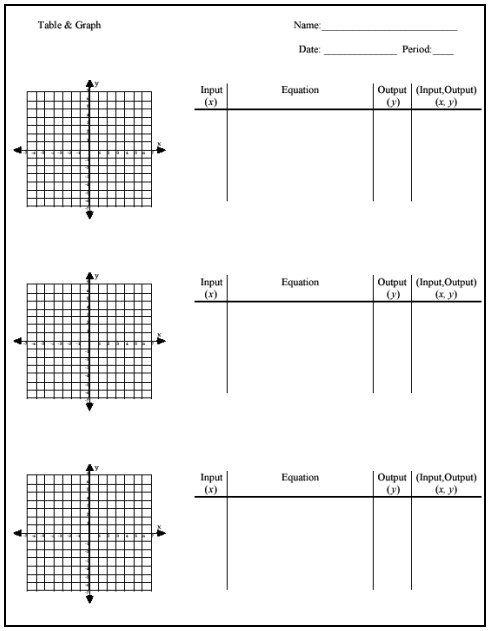
1) Since all lines are based on this equation we call this the "mother of all lines" or the "mother function". Point out to students that in mathematics $y = x$ is really not called the mother function.

2) Draw a point on the board/overhead. Ask, "How many lines can be drawn through that one point?" [infinite, show infinity symbol ∞]

3) Draw two distinct points on the board/overhead. Ask, "How lines can be drawn through those two points?" [one and only one]

4) "Thus, two distinct points determine a unique line. So we only need to find two distinct points to draw a line."

5) Pass out the graphing families black-line master (p. 21) shown below:



6) As a class, determine and substitute in two values for *x* and determine the correct values for *y*. Fill in the *t*-table, plot the two points, and draw the line.

7) Point out to students that every point on the line satisfies the equation $y = x$ and any point not on the line does not satisfy the equation $y = x$. Try a few ordered points on and not on the line to check.

8) Now hand out the graphing calculators. Instruct students not to do anything with the calculators until you give them specific instructions.

9) Have students slip off the covers and put the covers on the desk. Do not have students slip the covers back onto the calculators as the covers may get jammed.

10) Point out to students, using the overhead of the calculator face, the following important keys:

On	(bottom left key)
Y=	(top left key)
Graph	(top right key)
2^{nd} function	(colored key depending on calculator)
X, T, θ, n	(diagonal to 2^{nd} function key. Call it *x* key)
Arrows	(use as cursor key)
Clear	(below down arrow key)
+, -, x, ÷	(operation keys)
(-)	(negative key – refer to as such)

11) "Now we are going to graph the mother function – the mother of all lines - using the graphing calculator." Point out to that what they see on the overhead is what they should see on their calculator screens.

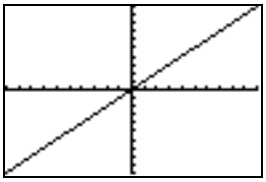
12) "Turn you calculators on."

13) "Press on the *Y*= key."

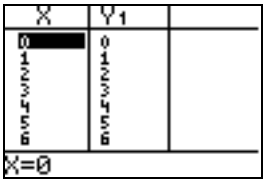
14) "Press on the *x* key"
(Note: that it is diagonal to the 2^{nd} function key)

15) "Press on the Graph key."

Students should see the following graph on their calculator:



16) Tell students that the calculator found values just like we did on our *t*-table, just faster. Using your overhead calculator show them the *t*-table by pressing on 2nd function and Graph. The screen will show a *t*-table such as:



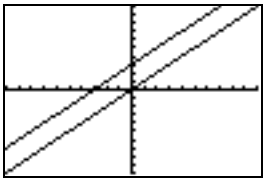
You can move through the *t*-table by using your up and down arrow keys. If students look at their *t*-tables you will need to have them press on the Graph key to get back to the graph.

Investigating $y = x + b$:

17) "Now we are going to look at what affect adding or subtracting a number has on the mother function. Let's try adding 3. Any ideas on what the graph will do when we add 3?" Have students make predictions before they enter it into the calculator.

18) "Press on *Y*=. Press the down arrow key one time to go to y_2. Press on *x* then press + 3. Now press the Graph key."

Students should see the following graphs on their calculator:

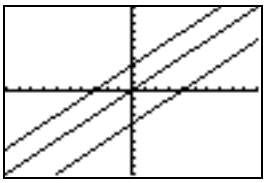


19) Summarize: "Adding 3 to the mother of all lines moved the line up 3 units. We call that a vertical phase shift."

20) "Based on this, do you have an idea of what the graph of $y = x - 4$ might look like? What happens to the mother of all lines when you subtract 4?" Have students make predictions before they enter it into the calculator.

21) "Press on *Y*=. Press the down arrow key two times to go to y_3. Press on *x* then press - 4. (Note: Press the subtraction key, not the negative key.) Now press the Graph key."

Students should see the following graphs on their calculator:



22) Summarize: "Subtracting 4 from the mother of all lines moved the line down 4 units. This is also called a vertical phase shift because we are moving the graph up or down.

Check for understanding:

A) "So what happens to the graph of $y = x + a$ when:

i) *a* is positive? [Moves the graph up]
ii) *a* is negative? [Moves the graph down]

B) Without students seeing (turn the overhead off), enter the graph $y = x + 7$. Now turn the overhead on and ask them to write the equation of the graph that is shown. "On the count of 3, read the equation you wrote down." [Y equals x plus 7]

Investigating $y = -x + b$:

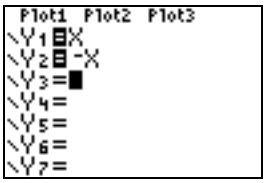
23) "Now we are going to investigate other members of the family of lines. Press *Y*=, press clear, and clear each of the equations shown on your screen. Note: you will need to use the up and down arrow keys to scroll through the equations so that you can clear them."

24) "Since we know what the *mother* of line ($y = x$) looks like, what do you think the opposite of the mother might look like ($y = -x$)?"

Have students show you with their hands what the mother looks like. Now have them do the opposite.

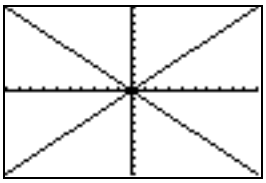
25) "Press on *Y*=. Enter the mother of all lines $y = x$. Press the down arrow key one time to go to y_2. Press the negative key. Press the *x* key.

Students should see the following screen on their calculator:



Then press the Graph key to graph both $y = x$ and $y = -x$.

Students should see the following graphs on their calculator:



26) Have students first predict then graph the following equations:

$$y = -x + 4$$
$$y = -x - 3$$

Check for understanding:

A) Describe the difference between the graph of $y = x$ and $y = -x$.

B) "So what happens to the graph of $y = -x + b$ when:

i) *b* is positive? [Moves the graph up]
ii) *b* is negative? [Moves the graph down]

C) Without students seeing (turn the overhead off), enter the graph $y = -x - 5$. Now turn the overhead on and ask them to write the equation of the graph that is shown. "On the count of 3, read the equation you wrote down." [Y equals -x minus 5]

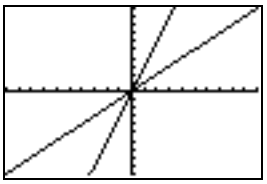
Investigating $y = mx$:

27) "Now we are going to investigate other members of the family of lines. Press *Y*=, press clear, and clear each of the equations shown on your screen. Note: you will need to use the up and down arrow keys to scroll through the equations so that you can clear them."

28) "What do you think is going to happen to the "mother" of all lines if we multiply the equation by a number?" Ask students for predictions of what the graph of $y = 3x$ might look like.

29) Have students enter in $y = x$ and then have them enter in $y = 3x$ Have them graph the equations.

Students should see the following graphs on their calculator:

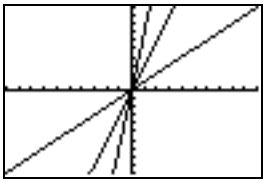


30) Have students describe in their own words what happened. Talk about/define the slope of a line.

31) Ask students what they could do to make the line even steeper.

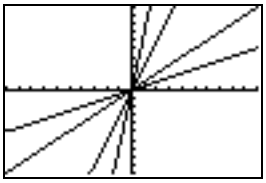
32) Have students graph $y = 7x$.

Students should see the following graphs on their calculator:



33) Ask students how they would make the graph less steep. Many students will say make it negative. Point out that the coefficient of the mother of all lines is 1 and the larger we made the coefficient the steeper the graph. "If we made it negative it would slope in the opposite direction. So we should try making the coefficient less than one and not negative. Let's try 0.5."

34) Have students enter in the equation $y = .5x$



Check for understanding:

A) Have students describe what happens to the graph of $y = mx + b$ when:

m is positive.
m is negative.
b is positive.
b is negative.

B) Turn the overhead off and enter in the following equation $y = -x + 3$
Press Graph.
Turn the overhead on and display the graph.
Ask students to work in pairs to determine the equation of the graph.

Do this several times for a variety of graphs.

C) Show students that what they have learned will help on standardized tests.

Investigating other Families of Functions – Quadratic Family

Follow a similar process to investigate the family of quadratics:

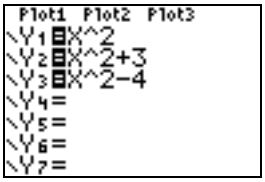
Investigating $y = x^2 + c$:

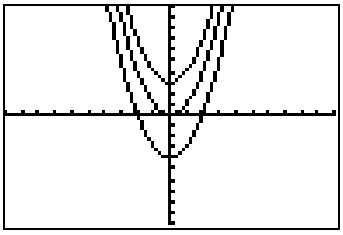
Note: I suggest you use the ^ key to raise to a power.

$y = x^2$

$y = x^2 + 3$

$y = x^2 - 4$



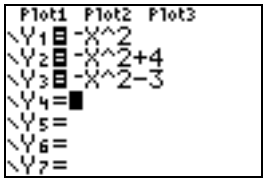


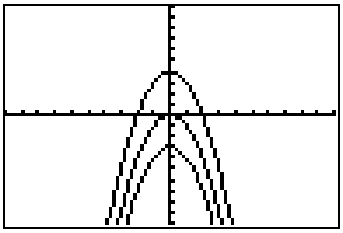
Investigating $y = -x^2 + c$:

$y = -x^2$

$y = -x^2 + 4$

$y = -x^2 - 3$





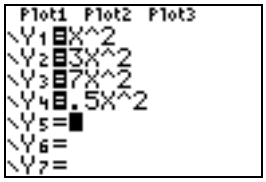
Investigating $y = ax^2$:

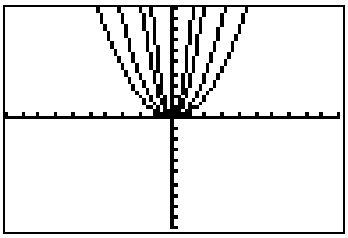
$y = x^2$

$y = 3x^2$

$y = 7x^2$

$y = .5x^2$





- **Check for understanding in a similar fashion to that of linear equations.**
- **You can investigate other families such as cubic, absolute value, etc.**

Possible Extension:

If you want to try to fool your students…

1) Tell them that the graphing calculators are amazing tools – almost like a small computer in your hand. The calculators have built-in sensors. Point out the "sensor" on the front left of the calculator.

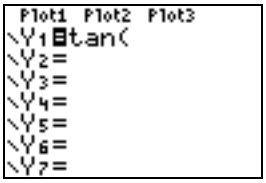
2) We are going to use this sensor to monitor your heart rate.

3) First have students press on *y*= and clear all.

4) Tell the students that they have to follow directions precisely or the heart monitor will not work properly.

5) Have them press the *TAN* key (it is to the left of the caret (^) key).

Students should see the following on their screens:



6) Have them press the *x* key.

7) Have them press the close parenthesis key) one time (it is below the *TAN* key).

Students should see the following on their screens

DO NOT PRESS GRAPH YET!

8) Tell the students to put their graphing calculators down. Tell them that in order to use the heart rate monitor we have to get our heart rates up. Tell them to stand up and run in place – do jumping jacks. Have them do this for several minutes encouraging them to have fun and get their heart rates up – a little aerobics in math class.

9) Now tell them that when you count to three, they are to sit down, turn on their calculators if necessary, put their index finger on the sensor and hit Graph. Be clear that it has to be the index finger – show them the index finger.

10) Begin to count … stretch it out …

1, …., 2, …. 4, ….

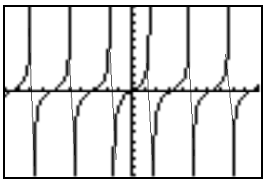
No really,

1, 2, 3

11) Remind them to sit down, put their index finger on the sensor, and hit Graph.

When they do …

Students should see something like the following on their screens:



12) The motion of the graph makes it appear as it might on a heart monitor.

13) Ask the students if you were running [No]. Press Graph on your calculator and move away from the graph.

14) Ask the students if you had your index finger on the "sensor". [No]

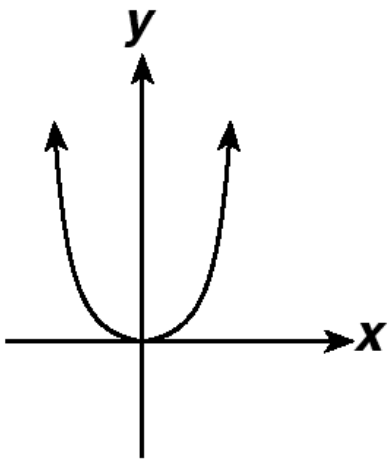
15) Let them know that it is not a heart monitor and they just got "played".

16) Tell them that this is why they should know more math – so they don't get "played".

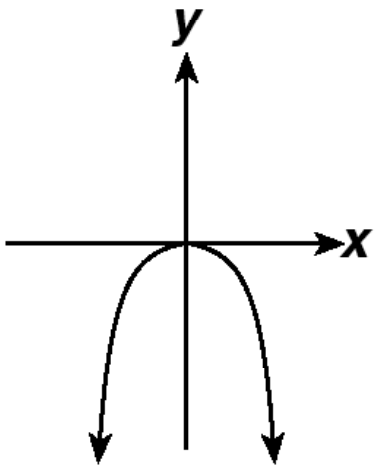
17) Let them know that *Tan* is another family of Trig functions – the tangent function – and that it is the graph of the "mother" of all tangent functions.

18) Show that now, after this lesson, students can easily answer the CST and CAHSEE released items (listed on the following pages).

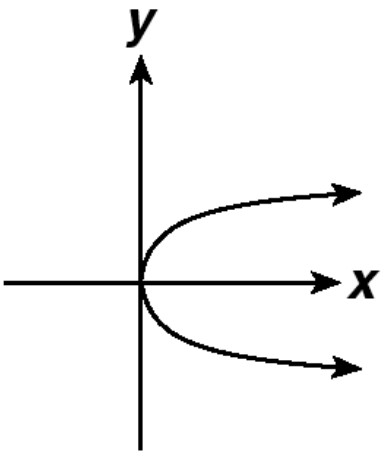
35 Which graph shows $y = -x^2$?



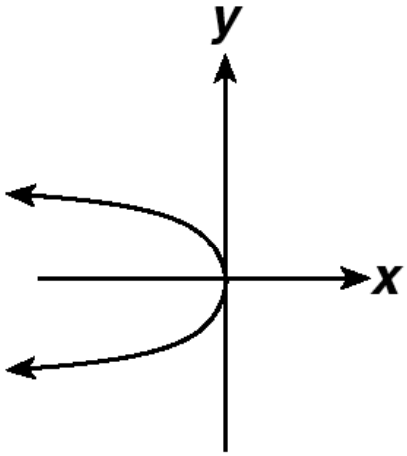
A



C

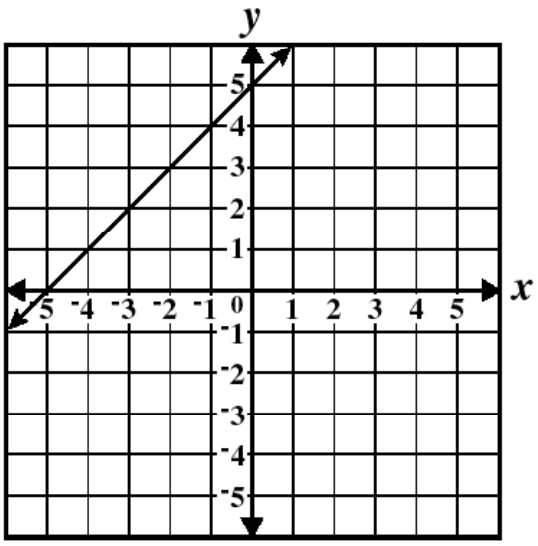


B

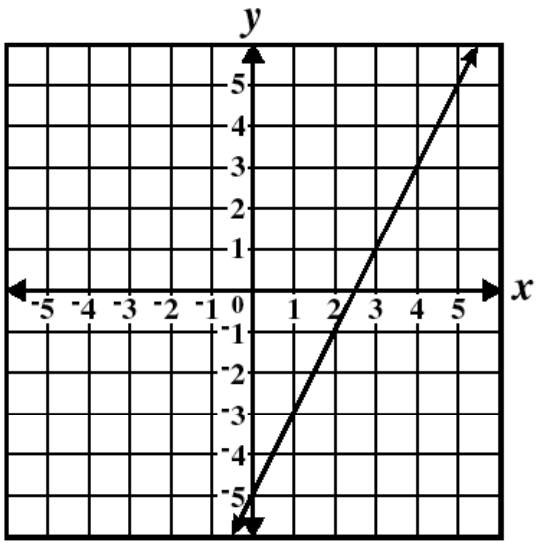


D

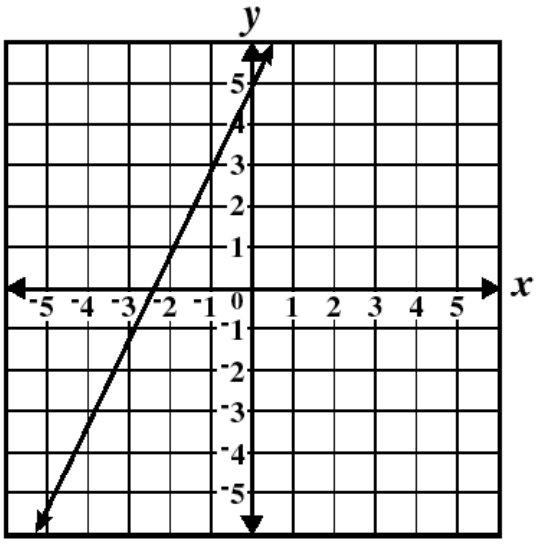
36 Which *best* represents the graph of $y = 2x - 5$?



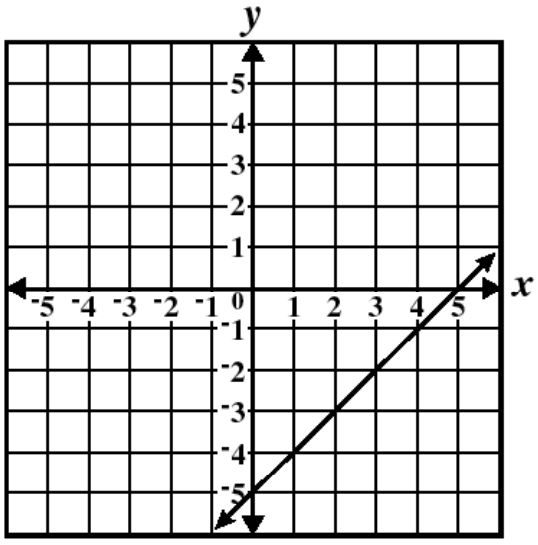
A



C



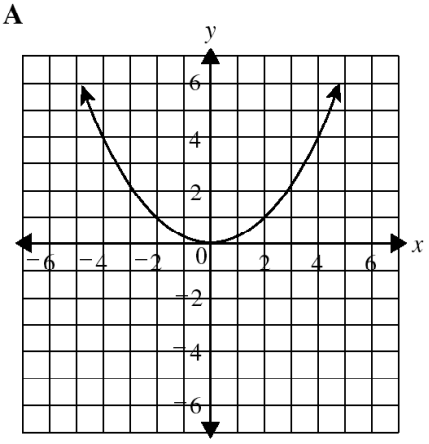
B



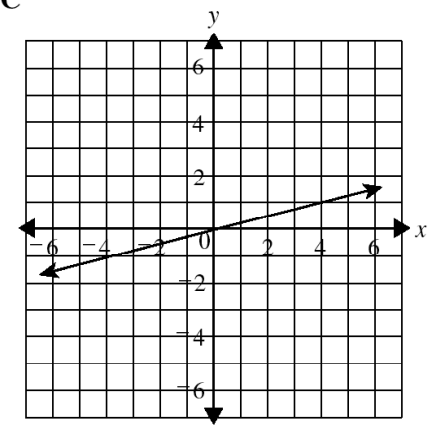
D

69. Which of the following is the graph of $y = \frac{1}{4}x^2$?

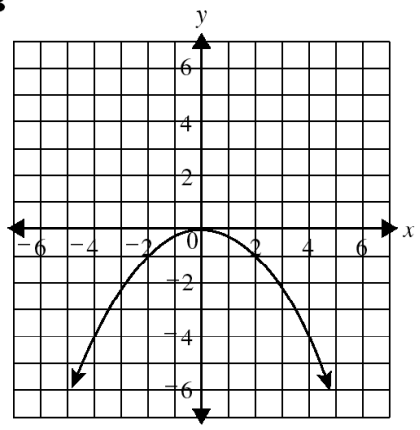
A



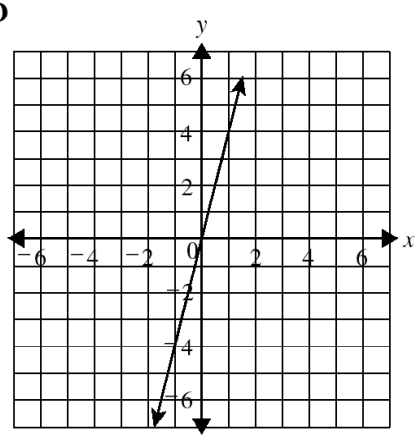
C



B



D



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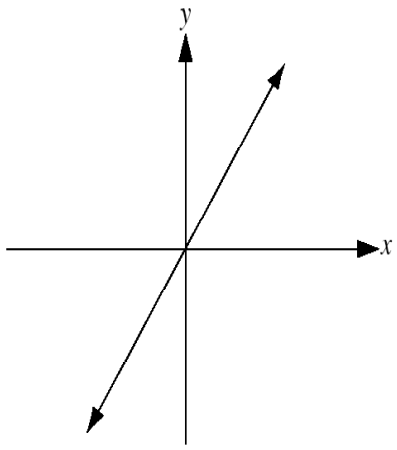
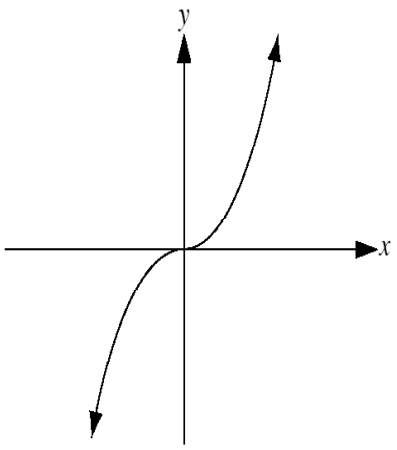
CAHSEE CST Released Item:

70. Which of the following could be the graph of $y = x^3$?

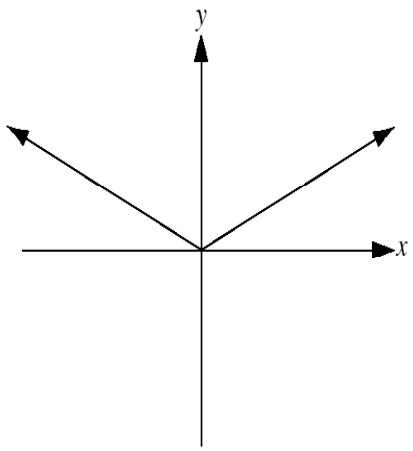
A



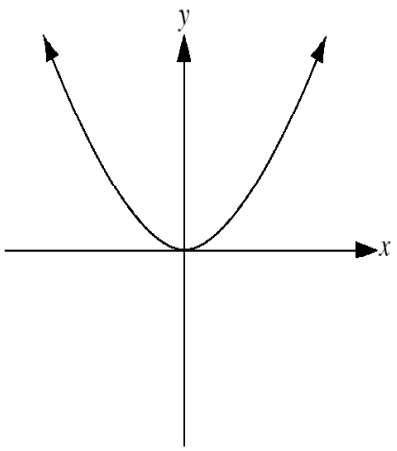
C



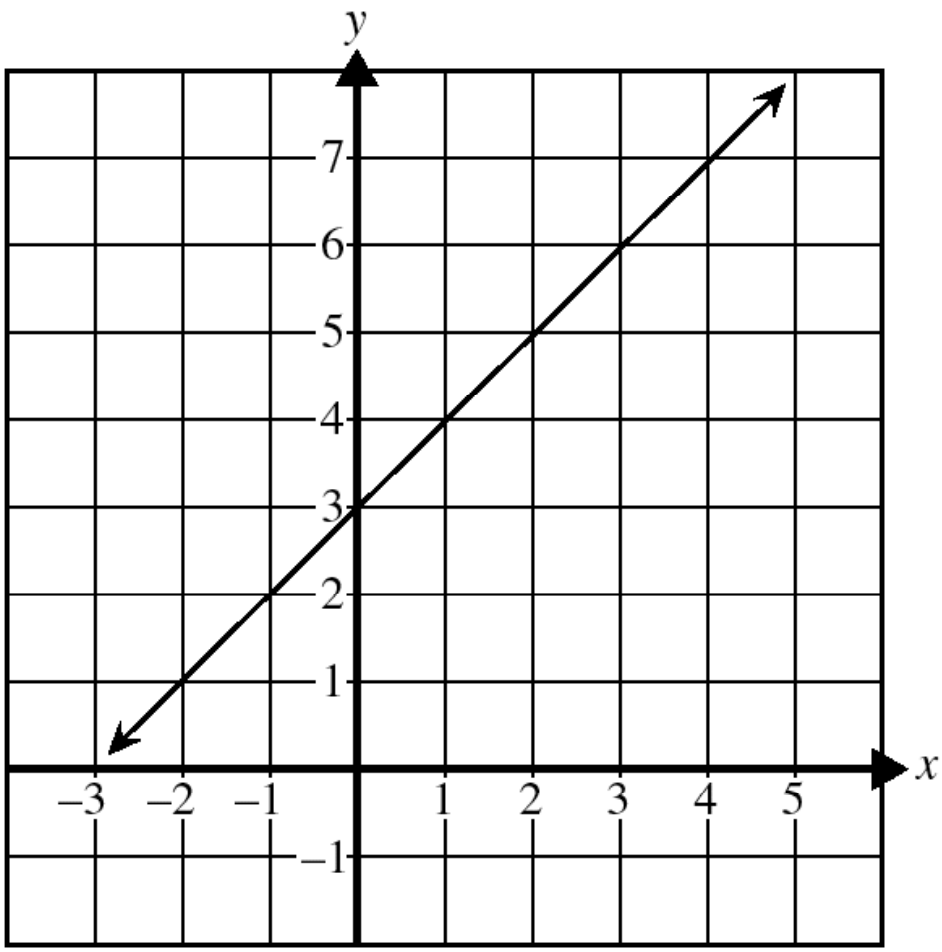
B



D



73. What is the equation of the graph shown below?

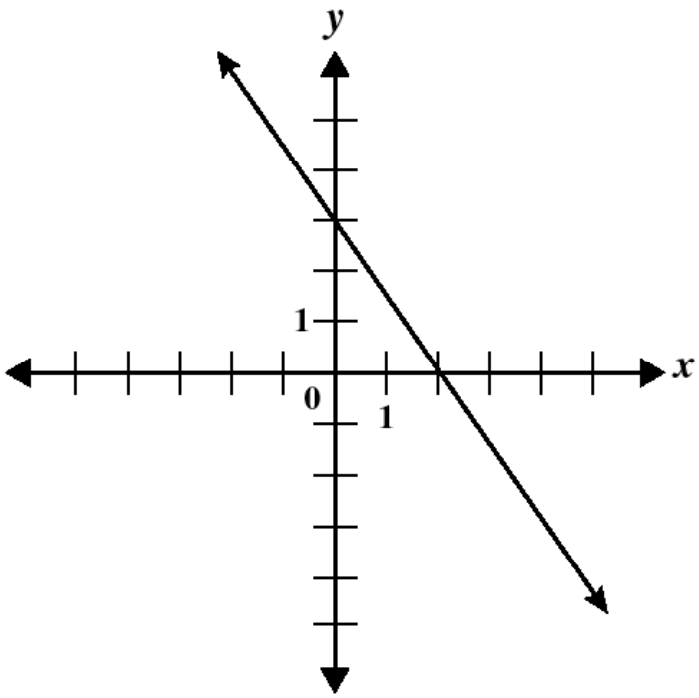


A $y = x - 1$

B $y = x + 1$

C $y = x + 3$

D $y = x - 3$



147. What is an equation of the line shown in the graph above?

A $y = -\frac{3}{2}x + 3$

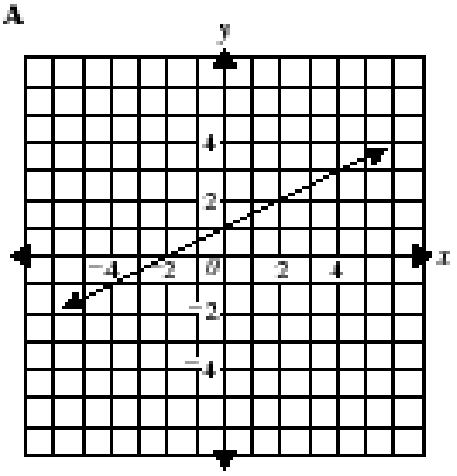
B $y = -\frac{2}{3}x + 2$

C $y = \frac{3}{2}x - 3$

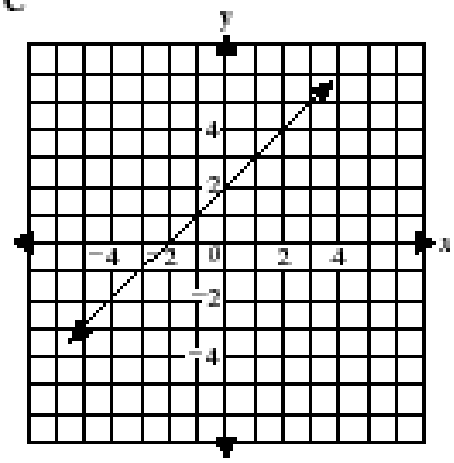
D $y = \frac{2}{3}x - 2$

148. Which of the following is the graph of $y = \frac{1}{2}x + 2$?

A



C



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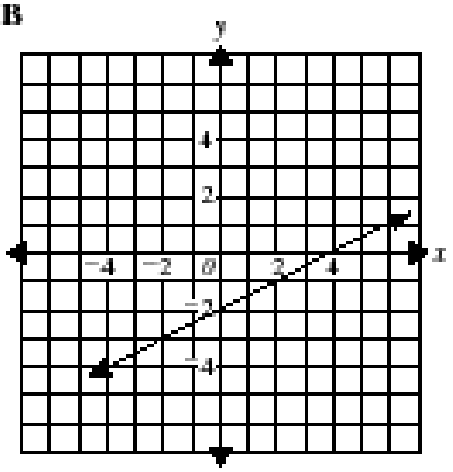
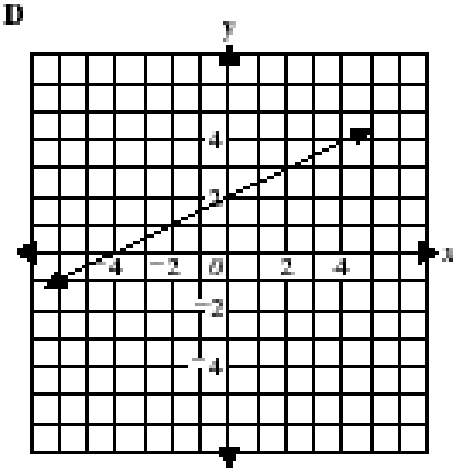


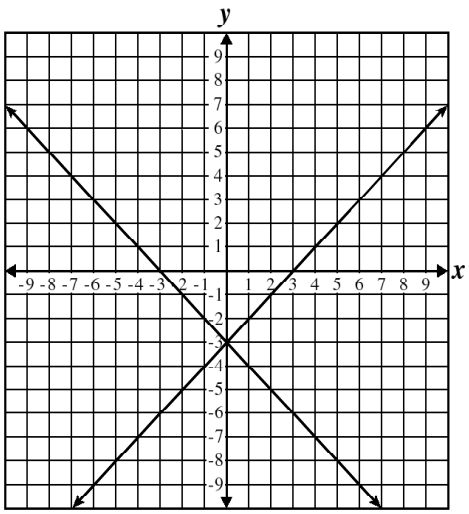
D



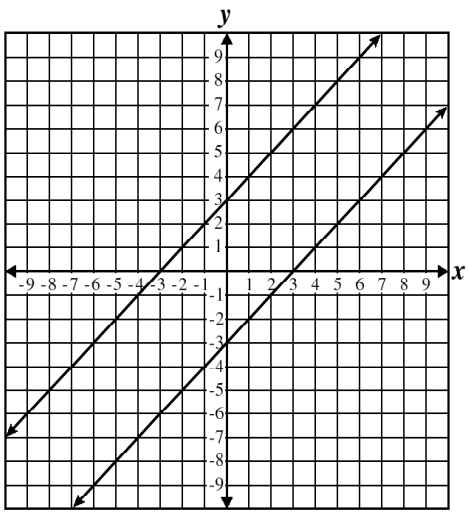
159. Which graph represents the system of equations shown below?

$$\begin{aligned} y &= -x + 3 \\ y &= x + 3 \end{aligned}$$

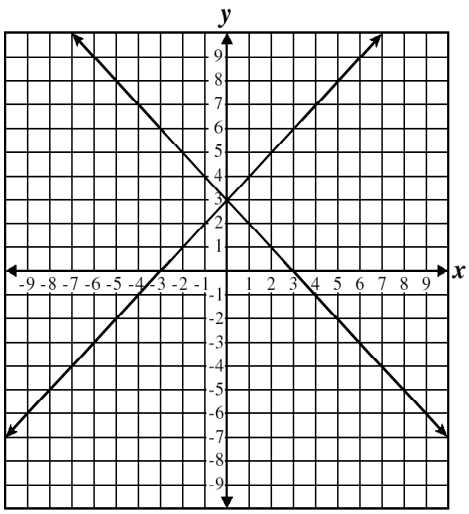
A



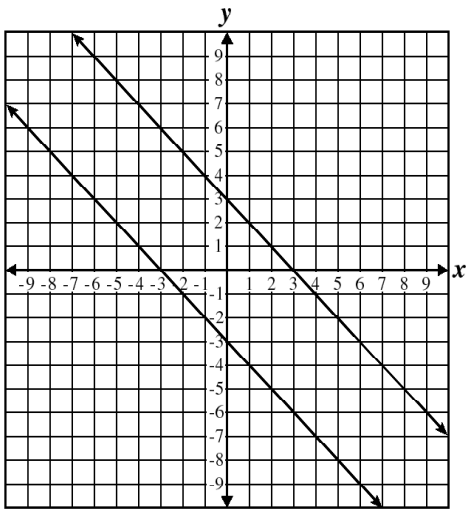
C



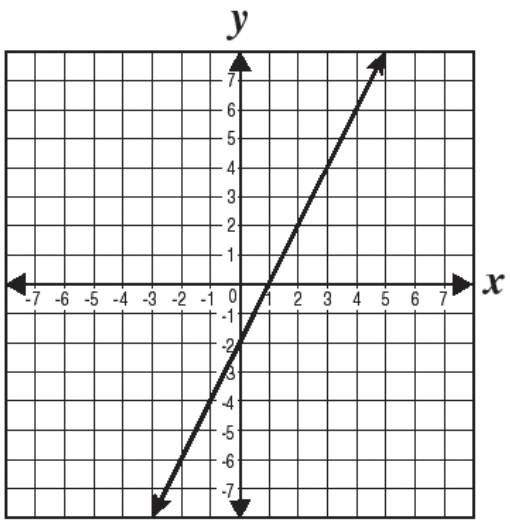
B



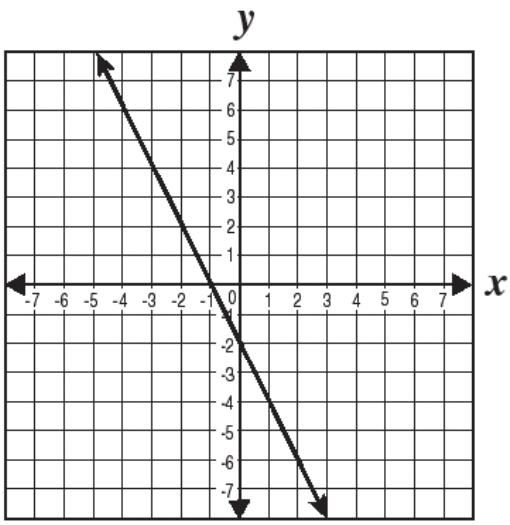
D



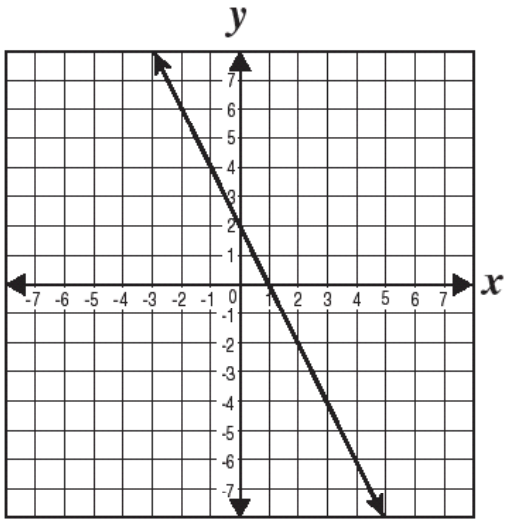
21 Which *best* represents the graph of $y = 2x - 2$?



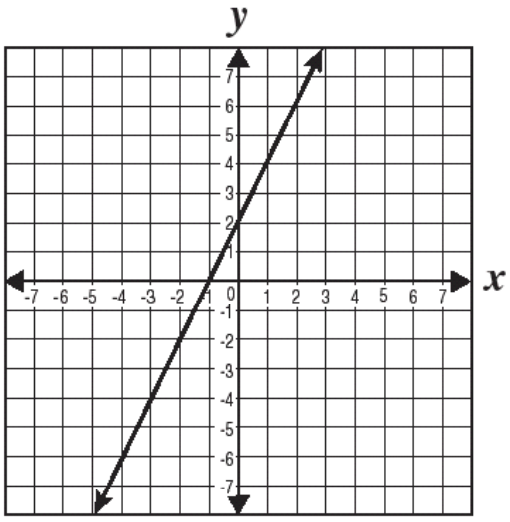
A



C



B

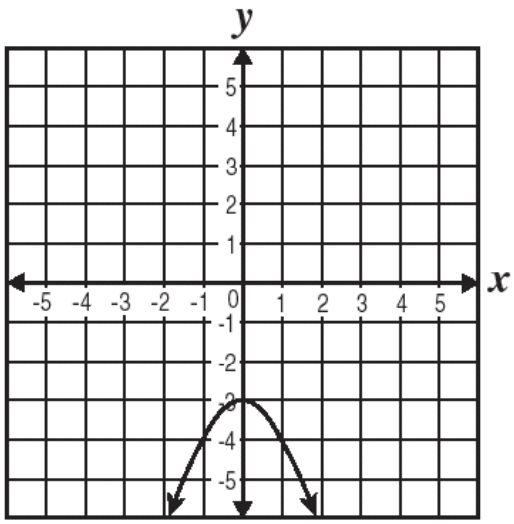


D

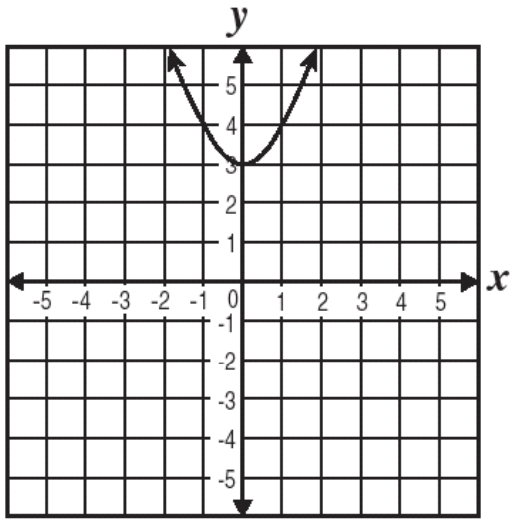
CSA00299

Algebra I CST Released Item:

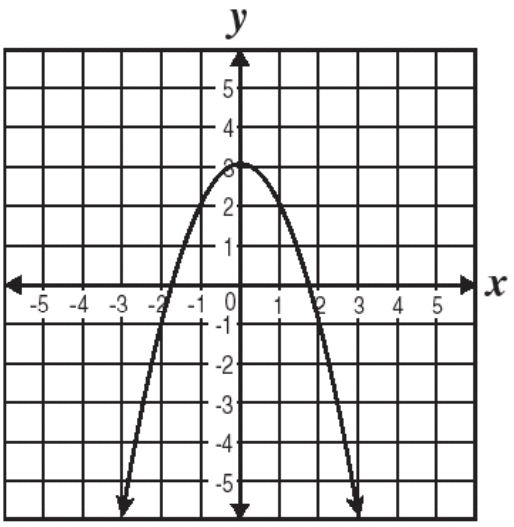
48 Which *best* represents the graph of $y = -x^2 + 3$?



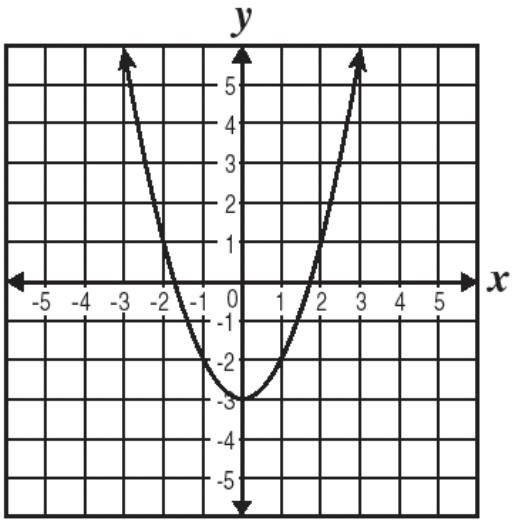
A



C



B



D

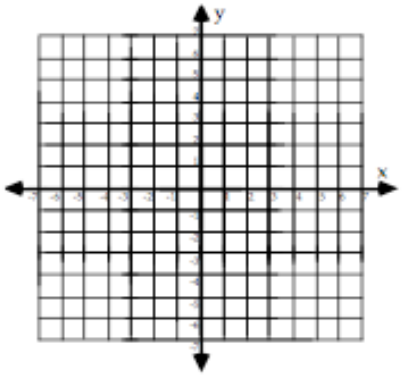
CSA00519

Table and Graph Master

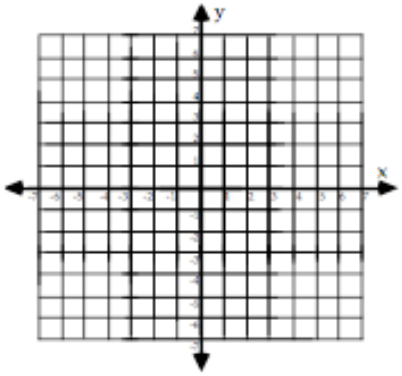
Table & Graph

Name:_____

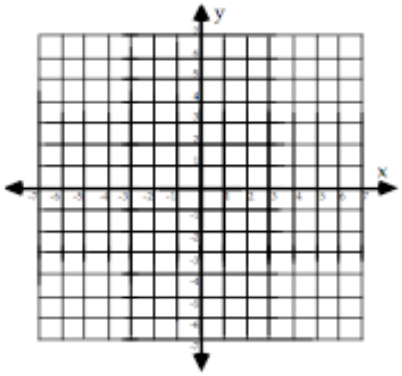
Date: _____ Period:____



Input (x)	Equation	Output (y)	(Input,Output) (x, y)



Input (x)	Equation	Output (y)	(Input,Output) (x, y)



Input (x)	Equation	Output (y)	(Input,Output) (x, y)